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Minimum Landing-Approach Distance for a Sailplane

Bion L. Pierson* and Imao Chen‡
Iowa State University, Ames, Iowa

Introduction

BECAUSE of the extremely low drag associated with modern high-performance sailplanes, the landing approach trajectory can become critical if aerodynamic deceleration devices are not used.[1] For the problem treated here, it is assumed that the sailplane approaches the landing strip head-on in still air with too much speed, altitude, or both to allow a conventional approach glide. It is also assumed that the initial altitude is too low for any kind of go-around maneuver. The problem can then be formulated as an optimal control problem, in which one seeks the lift coefficient time history which provides the shortest possible landing-approach distance. Alternately, the problem is one of transferring the sailplane from a prescribed initial state to a prescribed terminal state in a minimum distance. Furthermore, the flight is confined to a vertical plane. Sideslip or other lateral maneuvers are not allowed. The landing approach must also be made without benefit of spoilers, drag brakes, drag chutes, or other deceleration controls. Rotation (pitch) dynamics are neglected. Finally, it is necessary to impose minimum speed and altitude path constraints on the problem.

Problem Statement

Since the final time t_f is not specified, a control parameter, $\alpha = t_f$, is introduced via the time transformation

$$t=\alpha\tau \qquad 0\leq t\leq t_f \qquad 0\leq\tau\leq 1 \tag{1}$$

Thus, the variable end time problem will be transformed into a fixed end time problem with independent variable τ.

The point mass equations of motion are written with respect to the usual wind or trajectory axes.[2] Since the final range is to be minimized and since the range variable does not appear in the other dynamic equations, the range equation is simply incorporated into the performance index and is not required as part of the optimization process. The three remaining state variables are speed v, flight path angle γ, and altitude h.

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*Professor, Department of Aerospace Engineering and the Engineering Research Institute. Member AIAA.

†Graduate Student, Department of Aerospace Engineering.

The optimal control problem can be formally stated in terms of nondimensional variables as follows: Find the control function $u(\tau)$, $0\leq\tau\leq 1$ and the control parameter α which minimize the performance index

$$J=\alpha\int_0^1 v\cos\gamma d\tau + k_1^{-1}\int_0^1\left[(gX)^{1/2}\frac{v}{18}-1\right]^{-1}d\tau + k_2^{-1}\int_0^1 h^{-1}d\tau \tag{2}$$

subject to the dynamic constraints

$$\dot{v}=-\alpha[\eta C_D(u)v^2+\sin\gamma] \qquad v(0)=25(gX)^{-1/2} \tag{3a}$$

$$\dot{\gamma}=\alpha[\eta C_L(u)v^2-\cos\gamma]/v \qquad \gamma(0)=-0.02\text{rad} \tag{3b}$$

$$\dot{h}=\alpha v\sin\gamma \qquad h(0)=50/X \tag{3c}$$

and subject to the terminal state constraints

$$v(1)=23(gX)^{-1/2} \tag{4a}$$

$$\gamma(1)=0 \tag{4b}$$

$$h(1)=5/X \tag{4c}$$

where

$$C_D(u)=0.018556-0.009652C_L+0.022288C_L^2 \tag{5}$$

$$C_L(u)=C_{L_{\max.}}(2\sin^2 u-1) \tag{6}$$

and

$$\eta=\tfrac{1}{2}\rho gX/(mg/S)=0.01916015625X \tag{7}$$

Note that a quadratic drag polar, Eq. (5), has been adopted. The coefficients correspond to a hypothetical medium-performance sailplane with maximum lift-to-drag ratio slightly in excess of 32. Also, note that the use of the transformation, Eq. (6), insures a lift coefficient with magnitude less than $C_{L_{\max.}}=1.671$. From the boundary conditions in Eqs. (3) and (4), observe that the landing approach begins at an altitude of 50 m and a speed of 25 m/s and terminates at an altitude of 5 m and a speed of 23 m/s. Here, $X=1000$ m is an arbitrary characteristic length used in the nondimensionalization, and $g=9.81$ m/s^2 is the acceleration of gravity. In Eq. (3), the dot notation implies a derivative with respect to τ.

The second and third terms of the performance index, Eq. (2), represent integral interior penalty functions[3] for the speed and altitude path constraints, respectively. The second term limits the speed to values above the stall speed of 18 m/s. The third term enforces positive altitudes. As with any penalty function scheme, it is necessary to solve a sequence of unconstrained subproblems, Eqs. (2-7), with fixed positive penalty constants k_1 and k_2. These penalty constants are then increased between successive subproblems to allow the solution point to move closer to the active constraint surfaces. With the use of these interior penalty functions, it is necessary to begin computations with a nominal control which generates a trajectory satisfying both state inequality constraints.

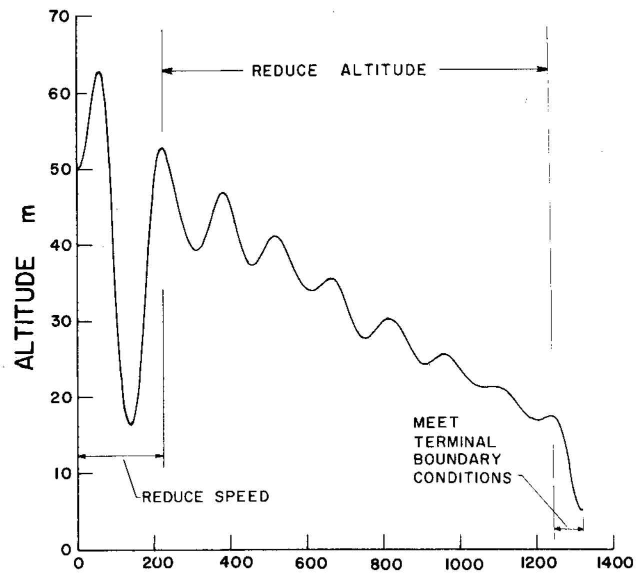

Fig. 1 Optimal trajectory.

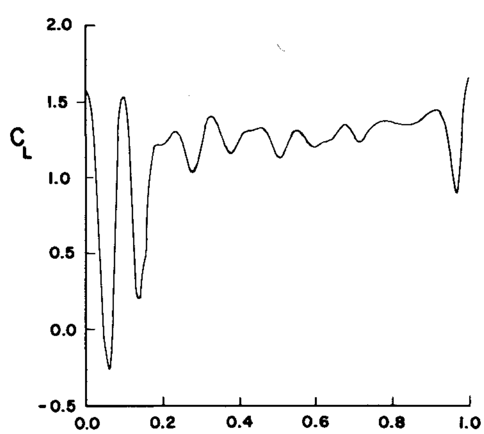

Fig. 2 Optimal lift coefficient time history.

Numerical Results

The optimal control problem just stated has been solved using a gradient projection algorithm[4] which incorporates conjugate directions of search for rapid convergence. The purpose of the projection operator in this algorithm is to obtain satisfaction of the terminal state constraints, Eq. (4), at each iteration. The method is a direct gradient method in that the control function $u(\tau)$ and the control parameter α are altered simultaneously on each iteration in an attempt to reduce J and satisfy the optimality condition.

All calculations were performed on coupled IBM 360/65 and AS/5 computers using a FORTRAN IV compiler and double-precision arithmetic. The required integrations were carried out using a standard fourth-order Runge-Kutta method with 100 fixed uniform integration steps. Three subproblems were solved. For each of these subproblems, the penalty constants k_1 and k_2 were each set equal to 200, 1000, and 5000, respectively. A final refinement run was made with $k_1 = 5000$, $k_2 = 10{,}000$, and 400 integration steps.

The optimal trajectory is shown in Fig. 1. The minimum landing-approach distance is 1317 m. The optimal final time is 65.09 s. The terminal state values for this trajectory agree with the values prescribed by Eq. (4) to at least six significant figures. Each peak on the optimal trajectory is associated with a near stall. The corresponding optimal lift coefficient (control) time history is presented in Fig. 2.

As may be noted in Fig. 1, the minimum-distance glide consists of three relatively distinct portions. Initially, the sailplane climbs. This is immediately followed by a steep dive and a pull-up to approximately the initial altitude. At this point, the speed has been reduced to almost the stall speed. There then follows a succession of shallow, but rapid, dives and climbs. These damped oscillations appear to converge to a straight-line trajectory with a glide slope of approximately 1/32, which in turn is the maximum L/D glide slope for this sail plane. A short final dive is required to match the specified terminal boundary conditions.

Discussion

The most obvious, and perhaps startling, feature of the optimal trajectory is its highly oscillatory nature. In practical terms, it is even questionable whether the trajectory can be flown, since the period of oscillation is only 7-8 s. Still, the solution provides useful qualitative information for high L/D aircraft. If the problem is viewed basically as one of energy dissipation in a viscous medium, then it seems reasonable to increase the total path length as much as possible. If sufficient lift is available, this implies an oscillatory trajectory. However, if the lift capability is severely reduced by the use of spoilers, for example, then one could anticipate that oscillatory trajectories would no longer be possible.

It should also be mentioned that this optimal control problem is a rather difficult one because of the multiple speed-constrained arcs present in the optimal trajectory. The lower bound on speed is necessary for the problem to have a solution. The altitude inequality constraint provides a realistic solution; without it, a solution with an impressive underground dive results. Related work on the application of optimal control methods to sailplane trajectory optimization problems may be found in Refs. 5 and 6.

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